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OPTIMAL PEREMPTORY CHALLENGES IN TRIALS BY JURIES: A BILATERAL --ETC(U)
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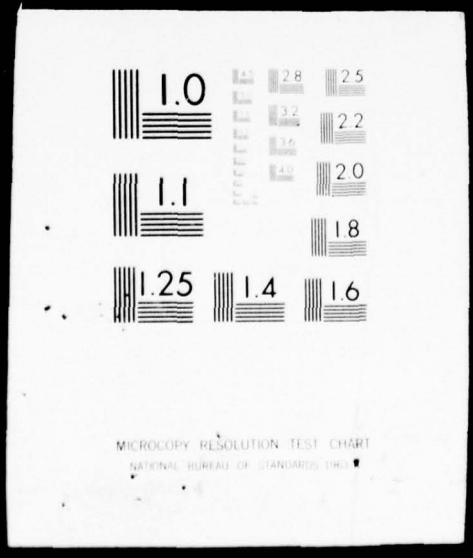

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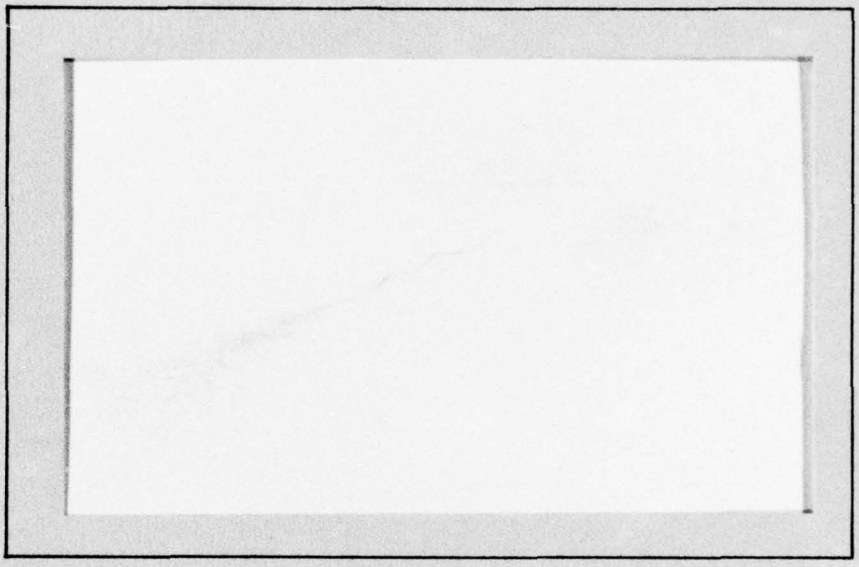

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OPTIMAL PEREMPTORY CHALLENGES IN TRIALS BY JURIES: A BILATERAL SEQUENTIAL PROCESS

by

Arthur Roth

Joseph B. Kadane

and

Morris DeGroot

Technical Report No. 122

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Optimal Peremptory Challenges in Trials by Juries:

A Bilateral Sequential Process

by Arthur Roth, Joseph B. Kadane and Morris DeGroot
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The application of social scientific methods of polling to the choice of which potential jurors to challenge peremptorilly has raised fears for the future of the jury system, as we now know it. Some of the cases in which these methods have been used include the Harrisburg Seven trial (Schulman et al. [8]),the Camden, New Jersey draft board raid trial, the Mitchell-Stans conspiracy trial (Arnold [1], Zeisel [10], and Zeisel and Diamond [11]), the Gainesville, Florida Veterans trial, the Wounded Knee, South Dakota trials of militant Indians and the Cedar Rapids murder trial arising from the Wounded Knee disturbance, the Buffalo Creek West Virginia dam disaster civil damage suit, the Ellsburg-Russo trial, the Joan Little murder trial, and the Attica trials (Shapley [9]). All of these trials involve highly publicized cases of defendants who have taken political positions likely to be very popular with some and very unpopular with others. Furthermore, the nature of the evidence in at least some of these trials was such as to confirm the prejudices of the jurors; especially in conspiracy trials, one man's conspiracy may be another man's business as usual.

To date the sophisticated methods have been used more extensively by the defense than by the prosecution (Kairys [7] and Ginger [6]). And it can be argued that this use is close to the intent of the jury system, to protect a defendant unpopular with his government

by having a group more politically diverse than the government decide his innocence or guilt.

The worry comes in the thought that now that the defense has blazed the trail, an overzealous prosecution, with the full financial resources of the government, may follow. If this occurs, one might conjure up images of "hanging juries" carefully chosen by sociological methods to have the most negative view of the defendant, and the defense, except in rare instances such as those discussed above, unable to match the resources of the government. "District Attorneys or U.S. Attorneys cannot be expected to stand by doing nothing while defendants in the most serious cases buy themselves a significant edge in trial after trial. The champions of the technique will have to realize that the days when it could be reserved for their favorite defendants will soon be over" (Etzioni [5]). Conceivably this could cause a threat to our civil liberties.

To examine whether this possible threat is to be taken seriously, one should first ask what the defense and prosecution would do with information of this type if they had it. In this paper we present a simplified model of the jury selection process and explore some of its implications. One of our difficulties in undertaking this work is that, while the law of most states is clear about the number of challenges allowed to the defense and prosecution in varying circumstances, the procedure is typically left to the trial judge. Usually the judge first examines potential jurors to be sure that they are qualified, and asks questions which might result in dismissal for cause, questions that vary depending on the nature of the trial. In our model each side then has the opportunity to peremptorilly challenge the next potential juror and, failing that, the juror is then

sworn in. The question of which side is to challenge first is left arbitrary in our mathematics, although in our model it cannot depend on previous uses of the peremptory challenge by either side. Furthermore we assume that the prosecution and the defense each have an opinion about whether the juror under consideration will vote for conviction, and that these opinions are known to each other.

This structure leads to a bilateral sequential process, in which decisions are made by each side one-by-one, without a simultaneous decision by the other side. Bilateral sequential processes may be a better model for many social phenomena, such as arms races and duopoly (Cyert and DeGroot [2,3]), than the traditional game theory that requires simultaneous moves by the players.

Both the information available to each side and the particular sequence we have chosen to study limit the applicability of this paper, and both assumptions need to be relaxed in further work. Nonetheless the particular structure we have chosen, although somewhat over-simplified, does represent a starting place for examining how effective sociological methods are likely to be.

1. Statement of the Problem and Main Results

Prosecution and defense lawyers are about to select a jury of J people. Each prospective juror is (sequentially) interviewed, and each lawyer must then decide whether to accept or challenge (i.e., reject) the present candidate before interviewing anyone else, and this decision cannot later be changed. The prosecution is

allowed at most A challenges while the defense has at most B of them. After questioning each juror, the two sides have (possibly different) opinions about the probability that this person will vote to convict the defendant, giving rise to a vector (p_{1i},p_{2i}) of the opinions of the prosecution and the defense, respectively. The joint c.d.f. $F(p_1,p_2)$ of the bivariate random variable (P_1,P_2) throughout the population is assumed known, so that the observed values (p_{1i},p_{2i}) represent a sequential random sample from F. It is also assumed (perhaps unrealistically) that after a juror is questioned, each side knows both its own and its opponent's opinion, i.e., the questioning process gives both sides simultaneously a complete (bivariate) observation from F. Furthermore, the rule determining at each stage which side must specify first whether it wishes to use a challenge is assumed fixed at the outset and does not depend on the previous decisions of the participants.

From the point of view of the prosecution at any stage in the selection process, the outcome of the entire process will be a random vector $(P_{11},P_{12},\ldots,P_{1J})$ of the p_{1i}-values of the members of the final jury. (Of course, the components of this vector become known one at a time as the selection process is carried out.) The prosecution has a utility function $U_1(P_{11},P_{12},\ldots,P_{1J})$, and he will attempt to maximize his expected utility at every stage. We assume that there is no interaction between jurors, so that the overall (random) probability of conviction in the opinion of the prosecution is $P^{(1)} = \prod_{i=1}^{J} P_{1i}$, where the product is taken over the J people on the final jury. (This assumption is probably valid only on the first post-trial ballot taken by the jury prior to any

discussion.) The prosecution's utility function can now be written as $U_1(P^{(1)})$. The analysis depends on the particular choice of this utility function (any increasing function is reasonable), and we proceed below using $U_1(P^{(1)}) = P^{(1)}$, so that the prosecution attempts to maximize $EP^{(1)}$. For the defense, we similarly define $P^{(2)} = \prod_{i=1}^{J} P_{2i}$ and denote the utility function U_2. The non-interaction assumption implies that the defense will maximize $E[U_2(P^{(2)})]$ at any stage, and any decreasing function is a reasonable choice for U_2. We use $U_2(P^{(2)}) = -P^{(2)}$, so that the defense attempts to <u>minimize</u> $EP^{(2)}$, the expected value of the overall probability of conviction in his opinion.

We show that an optimal (in a certain sense) strategy exists. We define our problem to be <u>reversible</u> (for our particular values of A, B, and J) if, under the optimal strategy, it will never matter at any stage which side is required to decide first whether or not to use a challenge. The problem is <u>universally reversible</u> if it is reversible for all possible values of A, B, and J. Both of these concepts depend on the joint c.d.f. F of P_1 and P_2. At any stage of the selection process, after some number of candidates have been acted upon (either mutually accepted or challenged by one side or the other), it is clear from our choice of utility functions that the problem is effectively beginning again with "new values" for A, B, and J. For any integers $a \leq A$, $b \leq B$, and $j \leq J$, we say that a,b, and j are <u>reachable</u> if there is positive probability using the optimal strategy that a,b, and j are ever these "new values". It is obvious that reversibility for A,B,J implies reversibility for any reachable a,b,j.

We specify an algorithm that finds the optimal strategy for both sides as a function of F. We find necessary and sufficient conditions on F under which the problem is reversible; obviously it is universally reversible if and only if these conditions hold for all A,B,J. In particular, universal reversibility is shown to hold whenever both sides always agree on the p-values of prospective jurors. We give examples of F's for which the problem is not universally reversible. We also show that each side can do at least as well by making the first decision regarding any juror as it can by having the opposition decide first.

2. Definition and Properties of the Optimal Procedure

Before investigating reversibility or finding the form of the optimal procedure, we must define this procedure and describe in what sense it is optimal. We observe that the jury must be selected after at most A + B + J people have been interviewed. Thus, the number of decisions in the selection process is bounded. Clearly the lawyer making the last possible decision (i.e., one juror remains to be selected, and this lawyer has one challenge remaining while his opponent has none) has an optimal choice. Under the assumption that this last possible choice will be made optimally, the consequences of the next-to-last possible decision are known. Hence it can also be made optimally. Proceeding by backward induction, each decision has an optimal choice if the side making that decision is willing to assume that both sides will act optimally on all subsequent decisions. The optimal procedure is taken to be the one resulting from all these optimal choices by both sides; it is optimal only in the sense of the assumptions just given. Since this procedure

completely defines the actions of both sides, it determines a pair of values $(EP^{(1)}, EP^{(2)})$, which represents the best the prosecution and the defense, respectively, can expect to do under the assumption that the other side will proceed optimally according to its own opinions about the prospective jurors.

3. Notation

Let $\underset{\sim}{w} = (w_1, w_2, \ldots, w_{A+B+J})$ be a vector such that $w_i = 1$ or 2 for $i = 1,2,\ldots,A+B+J$; $w_i = 1$ means that the prosecution has to decide first about the i^{th} candidate, while $w_i = 2$ means that the defense must decide first. For any vector $\underset{\sim}{y}$ with at least two elements, let $\emptyset(\underset{\sim}{y})$ denote the vector which is obtained by deleting the first element of $\underset{\sim}{y}$. For $a \leq A$, $b \leq B$, $j \leq J$, $p_1 \epsilon [0,1]$, and $p_2 \epsilon [0,1]$ suppose the prosecution has a challenges remaining, the defense has b of them, there are j jurors still to be selected, and the (p_{1i}, p_{2i}) associated with the present candidate is (p_1, p_2). In this situation, for $j = 1$ and 2, let $P^{(j')}$ be the product of the p_{ji}'s yet to be added to the jury, including the present candidate if he is accepted. Then we let $EP^{(1')}$ and $EP^{(2')}$ denote the expected values of these quantities under the optimal procedure described in section 2 above. Let $\underset{\sim}{v} = (v_1, v_2, \ldots, v_{a+b+j})$ be the vector consisting of the last $a + b + j$ elements of $\underset{\sim}{w}$, so that $\underset{\sim}{v}$ specifies who decides first for each remaining potential juror. Then we write $M^*(a,b,j,p_1,p_2,\underset{\sim}{v}) = EP^{(1')}$ and $M_*(a,b,j,p_1,p_2,\underset{\sim}{v}) = EP^{(2')}$ to show the explicit dependence of these quantities on the relevant parameters. We let $\mu^*(a,b,j,\underset{\sim}{v}) = EM^*(a,b,j,P_1,P_2,\underset{\sim}{v})$, where the joint distribution of (P_1,P_2) over the unit square has the c.d.f. F; $\mu_*(a,b,j,\underset{\sim}{v})$ is defined analogously. Of course, the quantities $\mu^*(a,b,j,\underset{\sim}{v})$ and $\mu_*(a,b,j,\underset{\sim}{v})$ represent the "values" of the remainder of the process to the two

sides prior to the interviewing of the candidate. Whenever a,b,j, and $\underset{\sim}{v}$ are not ambiguous, we shall conserve space by denoting $\mu^*(\alpha) = \mu^*(a-1,b,j,\emptyset(\underset{\sim}{v}))$, $\mu^*(\beta) = \mu^*(a,b-1,j,\emptyset(\underset{\sim}{v}))$, and $\mu^*(\gamma) = \mu^*(a,b,j-1,\emptyset(\underset{\sim}{v}))$; $\mu_*(\alpha),\mu_*(\beta)$, and $\mu_*(\gamma)$ denote the obvious analogues involving the μ_*-function. Also, let F_1 and F_2 denote the marginal c.d.f.'s of P_1 and P_2, respectively, both of which are easily obtained from the (known) joint c.d.f. F. Finally, let $\overline{F}(x,y) = 1 - F_1(x) - F_2(y) + F(x,y) = P(P_1 > x,\ P_2 > y)$.

4. The Form of the Optimal Strategy When Neither Side is Out of Challenges ($a \geq 1$, $b \geq 1$)

Case 1: Prosecution Makes the First Decision on the Next Candidate

When the prosecution makes the first decision on the next candidate, $v_1 = 1$, i.e., $\underset{\sim}{v}$ is of the form $\underset{\sim}{v} = 1\emptyset(\underset{\sim}{v})$. By considering the consequences of the two possible decisions, first for the prosecution and then (if the prosecution accepts the juror) for the defense, we can write (for $a \geq 1$, $b \geq 1$, $j \geq 1$)

$$(4.1)\quad M^*(a,b,j,p_1,p_2,1\emptyset(\underset{\sim}{v})) = \begin{cases} \max(\mu^*(\alpha), p_1\mu^*(\gamma)) & \text{if } p_2\mu_*(\gamma) < \mu_*(\beta) \\ \max(\mu^*(\alpha), \mu^*(\beta)) & \text{if } p_2\mu_*(\gamma) > \mu_*(\beta) \end{cases}.$$

It is obvious from the definitions that $\mu^*(\alpha) \leq \mu^*(\beta)$. Thus (4.1) can be rewritten (for $a \geq 1$, $b \geq 1$, $j \geq 1$) as

$$(4.2)\quad M^*(a,b,j,p_1,p_2,1\emptyset(\underset{\sim}{v})) = \begin{cases} \mu^*(\alpha) \text{ if } p_1 < \mu^*(\alpha)/\mu^*(\gamma) \text{ and } p_2 < \mu_*(\beta)/\mu_*(\gamma) \\ p_1\mu^*(\gamma) \text{ if } p_1 > \mu^*(\alpha)/\mu^*(\gamma) \text{ and } p_2 < \mu_*(\beta)/\mu_*(\gamma) \\ \mu^*(\beta) \text{ if } p_2 > \mu_*(\beta)/\mu_*(\gamma). \end{cases}$$

Note that the optimal decisions for both sides regarding the present juror can be deduced from (4.2); they are summarized in table (4.5) below. Using (4.2), we define p_1 to be "large" if $p_1 > \mu^*(\alpha)/\mu^*(\gamma)$ and "small" if $p_1 < \mu^*(\alpha)/\mu^*(\gamma)$; similarly, p_2 is "large" if $p_2 > \mu_*(\beta)/\mu_*(\gamma)$ and "small" if $p_2 < \mu_*(\beta)/\mu_*(\gamma)$. (These definitions depend on a,b,j, and $\underset{\sim}{v}$.) If the marginal distributions of P_1 and P_2 are both continuous, then p_1 and p_2 are each either large or small with probability one and table (4.5) completely describes the form of the optimal decisions. (Note that this can occur even if (P_1,P_2) does not have a jointly continuous distribution.) For present purposes, we assume the marginal distributions are continuous, so that the case $p_1 = \mu^*(\alpha)/\mu^*(\gamma)$ or $p_2 = \mu_*(\beta)/\mu_*(\gamma)$, which is treated in section 6 below, need not be considered here.

From table (4.5) below and by considering the proceedings from the standpoint of the defense, it can be easily seen that (for $a \geq 1$, $b \geq 1$, $j \geq 1$)

$$(4.3)\quad M_*(a,b,j,p_1,p_2,1\emptyset(\underset{\sim}{v})) = \begin{cases} \mu_*(\beta) \text{ if } p_2 \text{ is large} \\ \mu_*(\alpha) \text{ if } p_1 \text{ is small and } p_2 \text{ is small} \\ p_2\mu_*(\gamma) \text{ if } p_1 \text{ is large and } p_2 \text{ is small.} \end{cases}$$

Case II: Defense Makes the First Decision on the Next Candidate

When the defense decides first on the next candidate, $v_1 = 2$, i.e., $\underset{\sim}{v}$ is of the form $\underset{\sim}{v} = 2\emptyset(\underset{\sim}{v})$. By almost identical arguments to those used in Case I above, we can write (for $a \geq 1$, $b \geq 1$, $j \geq 1$)

$$(4.4)\quad M_*(a,b,j,p_1,p_2,2\emptyset(\underline{v})) = \begin{cases} \min(\mu_*(\beta), p_2\mu_*(\gamma)) & \text{if } p_1 \text{ is large} \\ \min(\mu_*(\beta), \mu_*(\alpha)) & \text{if } p_1 \text{ is small} \end{cases}$$

$$= \begin{cases} \mu_*(\beta) & \text{if } p_1 \text{ is large and } p_2 \text{ is large} \\ p_2\mu_*(\gamma) & \text{if } p_1 \text{ is large and } p_2 \text{ is small} \\ \mu_*(\alpha) & \text{if } p_1 \text{ is small} \end{cases}$$

since it follows from the definitions that $\mu_*(\alpha) \leq \mu_*(\beta)$. The optimal strategies now follow from (4.4), and we summarize these strategies <u>for both Case I and Case II</u>:

(4.5)

FIRST DECISION (Opponent may challenge if you accept)

p_1 \ p_2	small	large
small	defense: A prosecution: C	defense: A prosecution: A
large	defense: A prosecution: A	defense: C prosecution: A

FINAL DECISION (Opponent has already accepted)

p_1 \ p_2	small	large
small	defense: A* prosecution: C	defense: C prosecution: C
large	defense: A prosecution: A	defense: C prosecution: A*

A = accept, C = challenge, * = hypothetical case (opponent has already challenged)

Note that both sides' strategy does not depend on whether they are making the first or final decision <u>except when p_1 is small and p_2 is large</u> (i.e., both sides find the same juror undesirable). In that case, whoever decides first will accept the juror, forcing his opponent to be the one to use up a challenge. Table (4.5) gives the complete <u>form</u> of the optimal strategy when $a \geq 1$ and $b \geq 1$. It does not, however, tell us exactly what this strategy is because the

concepts "large" and "small" depend on a,b,j and $\underset{\sim}{v}$ through the functions μ^* and μ_* (with various sets of arguments), which we do not yet know how to evaluate (see section 7 below).

It follows from (4.5) that (for $a \geq 1$, $b \geq 1$, $j \geq 1$)

$$(4.6)\quad M^*(a,b,j,p_1,p_2,2\emptyset(\underset{\sim}{v})) = \begin{cases} \mu^*(\beta) \text{ if } p_1 \text{ is large and } p_2 \text{ is large} \\ p_1\mu^*(\gamma) \text{ if } p_1 \text{ is large and } p_2 \text{ is small} \\ \mu^*(\alpha) \text{ if } p_1 \text{ is small.} \end{cases}$$

5. The Desirability of Going First Plus a Characterization of Reversibility

From (4.2) and (4.6), we see that (for $a \geq 1$, $b \geq 1$, $j \geq 1$)

$$(5.1)\quad M^*(a,b,j,p_1,p_2,1\emptyset(\underset{\sim}{v})) - M^*(a,b,j,p_1,p_2,2\emptyset(\underset{\sim}{v})) = \begin{cases} \mu^*(\beta) - \mu^*(\alpha) \geq 0 \text{ if } p_1 \text{ small, } p_2 \text{ large} \\ 0 \qquad\qquad\qquad \text{otherwise.} \end{cases}$$

Similarly, from (4.3) and (4.4), for $a \geq 1$, $b \geq 1$, $j \geq 1$,

$$(5.2)\quad M_*(a,b,j,p_1,p_2,1\emptyset(\underset{\sim}{v})) - M_*(a,b,j,p_1,p_2,2\emptyset(\underset{\sim}{v})) = \begin{cases} \mu_*(\beta) - \mu_*(\alpha) \geq 0 \text{ if } p_1 \text{ small, } p_2 \text{ large} \\ 0 \qquad\qquad\qquad \text{otherwise.} \end{cases}$$

We see from (5.1) and (5.2) that both sides are at least as well off going first <u>for the next juror</u> as they are going second; there is no difference (in fact, we have seen from (4.5) that the strategies are independent of order) unless p_1 is small and p_2 is large. This argument can be extended by induction to other elements of the "order" vector $\underset{\sim}{v}$. Since reversibility is trivial if either $a = 0$ or $b = 0$ (i.e., only one side has any choices remaining), (5.1) and (5.2) suggest

<u>Theorem 1</u>: The optimal strategy is reversible if and only if for any reachable $a,b,j,\underset{\sim}{v}$ either (i) The probability is zero that p_1

is small and p_2 is large, or (ii) The probability is zero that p_1 and p_2 are either both small or both large not only for the present values of $a,b,j,\underset{\sim}{y}$ but also for any $a,b,j,\underset{\sim}{y}$ that are reachable from these present values.

Proof: The theorem would follow immediately from (5.1) and (5.2) if we could show that condition (ii) is equivalent to

(ii') $$\mu^*(\alpha) = \mu^*(\beta) \quad \text{and} \quad \mu_*(\alpha) = \mu_*(\beta).$$

But (ii') means that either side could give the other side one of its challenges without loss of utility. Since the μ^* and μ_*-functions represent expectations over the entire future of the selection process, (ii') is equivalent to the condition that (with probability one) it is not presently and will never in the future be the case that one side wants to challenge a candidate that the other side wants to accept. But this is precisely condition (ii). Q.E.D.

Theorem 1, unfortunately, is a characterization of reversibility that is as hard to verify as the original condition itself; hence the theorem has little practical use. In all the usual cases (where the defense and the prosecution have essentially opposite goals) it is clear that $\mu^*(\alpha) < \mu^*(\beta)$ for all $a,b,j,\underset{\sim}{y}$, and condition (ii) fails. If condition (ii) is ignored, then reversibility is equivalent to the property that P (both sides find the same juror unacceptable) = 0.

A very important special case of Theorem 1 is given by

Theorem 2: Suppose $P_2 = kP_1$ for some $k > 0$, i.e., the joint distribution of (P_1,P_2) lies entirely on a line through the origin. Then universal reversibility holds.

Proof: We can ignore the degenerate case where P_1 and P_2 are constants. Hence P_2 is a non-degenerate strictly increasing function of P_1, so that (see above remarks) condition (ii) of Theorem 1 cannot possibly be met, i.e., $\mu^*(\alpha) < \mu^*(\beta)$ for all $a,b,j,\underset{\sim}{v}$. For any values of the arguments, $M_*(a,b,j,p_1,p_2,\underset{\sim}{v}) = EP^{(2')} = k^j EP^{(1')} = k^j M^*(a,b,j,p_1,p_2,\underset{\sim}{v})$. Taking expectations with respect to P_1 and P_2, we obtain $\mu_*(a,b,j,\underset{\sim}{v}) = k^j \mu^*(a,b,j,\underset{\sim}{v})$. Assume for any $a,b,j,\underset{\sim}{v}$ that p_1 is not large and p_2 is not small. (In the discrete case, this may be a weaker assumption than p_1 small, p_2 large.) Then

(i) $p_1 \leq \mu^*(\alpha)/\mu^*(\gamma)$, i.e., $p_2 = kp_1 \leq k\mu^*(\alpha)/\mu^*(\gamma)$, and

(ii) $p_2 \geq \mu_*(\beta)/\mu_*(\gamma) = k^j\mu^*(\beta)/k^{j-1}\mu^*(\gamma) = k\mu^*(\beta)/\mu^*(\gamma) > k\mu^*(\alpha)/\mu^*(\gamma)$.

The result follows from Theorem 1 and the contradiction given by (i) and (ii).

Corollary: If both sides always agree on the p-value of any juror after questioning, then universal reversibility holds.

Note: Theorem 2 and its corollary have been proved even when P_1 or P_2 is marginally discrete. The fact that Theorem 1 also holds in the discrete case is a direct consequence of Theorem 3 below (which is proved without making use of Theorem 1).

6. The Case $p_1 = \mu^*(\alpha)/\mu^*(\gamma)$ or $p_2 = \mu_*(\beta)/\mu_*(\gamma)$

If p_1 or p_2 satisfies this condition, then one (or both) of these p-values is neither "large" nor "small". If one (or both) of

the marginal distributions of P_1 and P_2 is not continuous, this may happen with positive probability. When it does happen, the person whose p-value is neither large nor small will be indifferent between his two possible decisions. In that case, the decision his opponent would prefer is called the benevolent decision while the other decision is called the malevolent decision. A lawyer who always makes the benevolent decision when he is indifferent (whether he is deciding first or whether his opponent has already accepted the juror) is said to adopt the benevolent strategy; the malevolent strategy is defined analogously. (Of course, it is possible for a lawyer to make some benevolent decisions and some malevolent ones, but we will not consider such "mixed" strategies.)

Theorem 3: If the benevolence or malevolence of one lawyer's strategy is known to his opponent, the benevolence or malevolence may affect the opponent's strategy but will not alter the presence or absence of reversibility or universal reversibility.

Proof: Suppose that the defense is indifferent on a particular decision. If the prosecution is also indifferent, the result is trivial. Hence assume that this is not the case, i.e., no two of $\mu^*(\alpha)$, $\mu^*(\beta)$, and $p_2\mu^*(\gamma)$ are equal. Then the following table covers all possible cases and is easy to derive:

DEFENSE BENEVOLENT

	defense first	prosecution first
$\mu^*(\beta) < p_2\mu^*(\gamma)$	both accept	both accept
$\mu^*(\beta) > \mu^*(\alpha) > p_2\mu^*(\gamma)$	defense challenge	prosecution accept / defense challenge
$\mu^*(\beta) > p_2\mu^*(\gamma) > \mu^*(\alpha)$	defense challenge	prosecution accept / defense challenge

DEFENSE MALEVOLENT

	defense first	prosecution first
$\mu^*(\beta) < p_2\mu^*(\gamma)$	defense challenge	prosecution accept / defense challenge
$\mu^*(\beta) > \mu^*(\alpha) > p_2\mu^*(\gamma)$	defense accept / prosecution challenge	prosecution challenge
$\mu^*(\beta) > p_2\mu^*(\gamma) > \mu^*(\alpha)$	both accept	both accept

The action taken regarding this juror is seen to be independent of order. However, from the second and fifth lines of the above table, the prosecution's strategy is seen to depend on the benevolence or malevolence of the defense.

A similar analysis when the prosecution is indifferent completes the proof.

7. An Algorithm for Determining the Optimal Procedure

The form of the optimal procedure (so long as $a \geq 1$, $b \geq 1$) was found in section 4. To completely specify the procedure, it remains

only to evaluate the functions μ^* and μ_* (in order to quantify the notions of "large" and "small" values for p_1 and p_2). For any two real numbers s and t, define the set

$$(7.1) \qquad S(s,t) = \{(x,y) : x > s \text{ and } y < t\}.$$

Then for any bivariate c.d.f. $\Delta(x,y)$, define the two transformations

$$(7.2)\ U_\Delta(s,t) = \iint_{S(s,t)} x\,d\Delta(x,y) \left[= \int_s^\infty \int_{-\infty}^t x\delta(x,y)\,dy\,dx \text{ if } \Delta \text{ has a density } \delta \right],$$

$$(7.3)\ V_\Delta(s,t) = \iint_{S(s,t)} y\,d\Delta(x,y) \left[= \int_s^\infty \int_{-\infty}^t x\delta(x,y)\,dy\,dx \text{ if } \Delta \text{ has a density } \delta \right].$$

Our bivariate c.d.f. $F(p_1,p_2)$ represents a distribution on the unit square, and hence the quantities $-\infty$ and ∞ in (7.2) and (7.3) can be replaced by 0 and 1, respectively, when evaluating U_F and V_F. Also, $U_F(s,t) = V_F(s,t) = 0$ if either $s > 1$ or $t < 0$; $U_F(s,t) = U_F(0,t)$ and $V_F(s,t) = V_F(0,t)$ if $s < 0$; $U_F(s,t) = U_F(s,1)$ and $V_F(s,t) = V_F(s,1)$ if $t > 1$. Taking expectations on both sides of (4.2), (4.3), (4.4), and (4.6), respectively, we now obtain (for $a \geq 1, b \geq 1, j \geq 1$) the relationships

$$(7.4)\ \mu^*(a,b,j,1\emptyset(\underset{\sim}{y})) = \mu^*(\alpha)F\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right) + \mu^*(\beta)\left[1-F_2\left(\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right)\right] +$$

$$\mu^*(\gamma)U_F\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right),$$

$$(7.5)\quad \mu_*(a,b,j,1\emptyset(\underset{\sim}{v})) = \mu_*(\alpha)F\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right) + \mu_*(\beta)\left[1-F_2\left(\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right)\right] +$$

$$\mu_*(\gamma)V_F\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right),$$

$$(7.6)\quad \mu_*(a,b,j,2\emptyset(\underset{\sim}{v})) = \mu_*(\alpha)F_1\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\right) + \mu_*(\beta)\bar{F}\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right) +$$

$$\mu_*(\gamma)V_F\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right), \text{ and}$$

$$(7.7)\quad \mu^*(a,b,j,2\emptyset(\underset{\sim}{v})) = \mu^*(\alpha)F_1\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\right) + \mu^*(\beta)\bar{F}\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right) +$$

$$\mu^*(\gamma)U_F\left(\frac{\mu^*(\alpha)}{\mu^*(\gamma)}\,\frac{\mu_*(\beta)}{\mu_*(\gamma)}\right),$$

where we recall that $\mu^*(\alpha) = \mu^*(a-1,b,j,\emptyset(\underset{\sim}{v}))$, $\mu^*(\beta) = \mu^*(a,b-1,j,\emptyset(\underset{\sim}{v}))$, $\mu^*(\gamma) = \mu^*(a,b,j-1,\emptyset(\underset{\sim}{v}))$, and $\mu_*(\alpha),\mu_*(\beta)$, and $\mu_*(\gamma)$ are defined analogously. Hence (7.4) through (7.7) define a recursive formula for μ^* and μ_* in terms of μ^*-functions and μ_*-functions of lower order as well as F, U_F, and V_F. The algorithm defined by (7.4) through (7.7) merely requires a set of boundary conditions to completely determine μ^* and μ_* for all possible arguments. The

boundary conditions are (for arbitrary $\underset{\sim}{v}$)

(7.8) $\mu^*(a,b,0,\underset{\sim}{v}) = \mu_*(a,b,0,\underset{\sim}{v}) = 1$ for any a,b,

(7.9) $\mu^*(a,0,j,\underset{\sim}{v}) = u^*(a,j)$ for $a \geq 1$, $j \geq 1$,

(7.10) $\mu_*(a,0,j,\underset{\sim}{v}) = u_*(a,j)$ for $a \geq 1$, $j \geq 1$,

(7.11) $\mu_*(0,b,j,\underset{\sim}{v}) = v_*(b,j)$ for $b \geq 1$, $j \geq 1$, and

(7.12) $\mu^*(0,b,j,\underset{\sim}{v}) = v^*(b,j)$ for $b \geq 1$, $j \geq 1$,

where u^* and v_* represent the one-sided versions of this problem, and where u_* and v^* are the values the "non-players" in these one-sided versions can expect by helplessly watching their opponents carry out their strategy. Separate algorithms for evaluating these four functions are given below.

Before generating the algorithms for u^*, u_*, v^*, and v_*, we note that it is clear that u^* depends only on $F_1(p_1)$ and v_* depends only $F_2(p_2)$, but u_* and v^* depend on the entire joint distribution of P_1 and P_2. In preparation for dealing with the two marginal (univariate) c.d.f.'s, we define for any univariate c.d.f. Λ the transformation

(7.13) $T_\Lambda(s) = \int_s^\infty (x-s)d\Lambda(x)$ [$= \int_s^\infty (x-s)\lambda(x)dx$ if Λ has a density λ].

The properties of this transformation are detailed in section 11.8 of DeGroot [4]. Of course, since P_1 and P_2 are random variables on [0,1], we can replace ∞ by 1 in (7.13) when evaluating T_{F_1} and T_{F_2}. Suppose that X is a random variable with c.d.f. G, that $Y = DX$ for some constant $D > 0$, and that H is the c.d.f.

of Y. Then for any constant K, it is easily shown that

(7.14) $E[\max(K,X)] = K + T_G(K)$,

(7.15) $E[\min(K,X)] = EX - T_G(K)$, and

(7.16) $T_H(s) = DT_G(\frac{s}{D})$.

Algorithms for u^* and v_* are now easily obtained using (7.14) through (7.16):

(7.17) $$u^*(a,j) = E\{\max[u^*(a-1,j), P_1 u^*(a,j-1)]\} = u^*(a-1,j) + u^*(a,j-1)T_{F_1}\left(\frac{u^*(a-1,j)}{u^*(a,j-1)}\right),$$

(7.18) $$v_*(b,j) = E\{\min[v_*(b-1,j), P_2 v_*(b,j-1)]\} = v_*(b,j-1)\cdot EP_2 - v_*(b,j-1)T_{F_2}\left(\frac{v_*(b-1,j)}{v_*(b,j-1)}\right).$$

The appropriate boundary conditions for (7.17) and (7.18), as well as for the functions u_* and v^*, are

(7.19) $u^*(a,0) = u_*(a,0) = v_*(b,0) = v^*(b,0) = 1$ for any a or b,

(7.20) $u^*(0,j) = v^*(0,j) = (EP_1)^j$ for any $j \geq 1$, and

(7.21) $u_*(0,j) = v_*(0,j) = (EP_2)^j$ for any $j \geq 1$.

Algorithms for u_* and v^* are more difficult to obtain. To compute $u_*(a,j)$, for example, we note from (7.17) that the prosecution will challenge the next juror if and only if $P_1 < u^*(a-1,j)/u^*(a,j-1) = Q$, say. Then

(7.22) $u_*(a,j) = EW$, where

(7.23) $$W = \begin{cases} u_*(a-1,j) \text{ if } P_1 < Q \\ P_2 u_*(a,j-1) \text{ if } P_1 > Q. \end{cases}$$

Combining (7.22) and (7.23), we obtain

$$(7.24)\quad u_*(a,j) = u_*(a-1,j)F_1(Q) + u_*(a,j-1)\int_Q^1 E(P_2|P_1=p_1)dF_1(p_1)$$

$$= u_*(a-1,j)F_1(Q) + u_*(a,j-1)\int_Q^1\int_0^1 p_2 dF(p_2|p_1)dF_1(p_1)$$

$$= u_*(a-1,j)F_1(Q) + u_*(a,j-1)V_F(Q,1)$$

since $dF(p_2|p_1)\cdot dF_1(p_1) = dF(p_1,p_2)$. By identical methods, one can obtain

$$(7.25)\quad v^*(b,j) = v^*(b-1,j)[1-F_2(R)] + v^*(b,j-1)U_F(0,R)$$

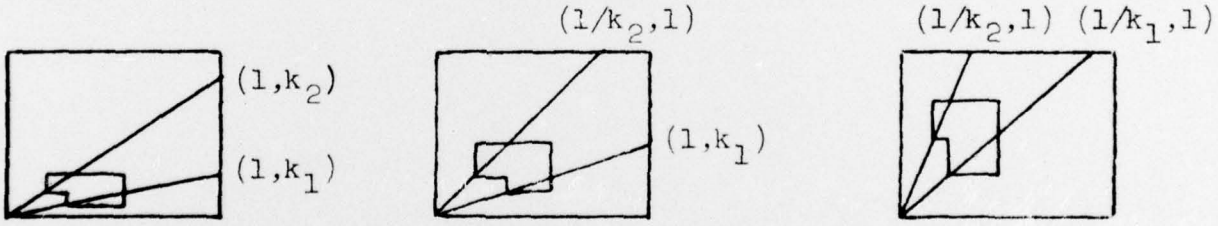
where $R = v_*(b-1,j)/v_*(b,j-1)$. Equations (7.17), (7.18), (7.24), and (7.25), together with the boundary conditions (7.19) to (7.21), form complete algorithms for evaluating the original boundary conditions (7.9) to (7.12). The functions u^* and u_* can now be computed for any arguments, and the optimal procedure is completely specified for $a \geq 1$ and $b \geq 1$. When $a = 0$ and $b \geq 1$, the defense is playing a one-sided game and we see from (7.18) that the optimal strategy is to challenge the next juror if and only if $P_2 > R$. Similarly, when $b = 0$ and $a \geq 1$, we have already seen from (7.17) that the best strategy for the prosecution is to challenge if and only if $P_1 < Q$. When $a = b = 0$, no strategy at all is involved. The entire optimal strategy has now been specified.

8. Examples That Are Not Universally Reversible

Example 1: It follows directly from Theorem 1 that reversibility cannot hold (for any a,b,j at all) if P_1 and P_2 are independent (and neither is a constant).

Example 2: Since the problem is universally reversible if $P_2 = kP_1$, we might suspect that this is also the case when the distribution of (P_1,P_2) lies on the union of two such straight lines, i.e., $P_2 = k_1P_1$ (denote this line L_1) or $P_2 = k_2P_1$ (denoted L_2). However, we show that some F's that are not universally reversible satisfy this condition. Without loss of generality, assume $0 < k_1 < k_2$. The three possible cases (depending on how k_1 and k_2 compare to 1) are illustrated below:

Case 1: $k_2 \leq 1$ Case 2: $k_1 \leq 1 < k_2$ Case 3: $k_1 > 1$



Let $T(k_1,k_2)$ be a subset of the unit square with the following property: If $(x_1,y_1) \in L_1 \cap T(k_1,k_2)$ and $(x_2,y_2) \in L_2 \cap T(k_1,k_2)$, then $x_1 \geq x_2$. (Such a set can be found for any $0 < k_1 < k_2$ -- see enclosed areas in the above diagrams.) Suppose the distribution of (P_1,P_2) lies entirely on $T(k_1,k_2) \cap (L_1 \cup L_2)$. Suppose furthermore that very little of the probability lies on L_2 (and hence most of it lies on L_1). Then for relatively close values of a and b, it is clear that any juror whose (p_1,p_2) lies on L_2 will

be unsatisfactory to both sides. Lack of universal reversibility follows from Theorem 1.

Comment: We cannot "fix" the above by requiring that the support of the distribution of (P_1,P_2) be all of $L_1 \cup L_2$. In that case, we can construct essentially the same example by putting an exceedingly negligible amount of the probability on $(L_1 \cup L_2) \cap [T(k_1,k_2)]^c$.

Conjecture for an Example 3: If the distribution of (P_1,P_2) is absolutely continuous with respect to Lebesque measure on the plane, then the optimal strategy is not universally reversible.

9. Two Numerical Examples With the Same Marginal Distributions

Suppose P_1 and P_2 each have (marginal) uniform distributions on $[0,1]$ and that $A = B = J = 1$ (i.e., one juror is to be selected and each side has one challenge). We compute the relevant results in the two cases where (i) $P_1 = P_2$ and (ii) P_1 and P_2 are independent.

Example 1: $P_1 = P_2$. By either Theorem 2 or its corollary, we have universal reversibility. By the proof of Theorem 2, $M^* = M_*$ and $\mu^* = \mu_*$ for any possible common arguments. Furthermore, we can write $M^*(a,b,j,p,\underset{\sim}{v})$ since p_1 and p_2 will always be the same. Thus

$$(9.1)\quad M^*(1,1,1,p,1\emptyset(\underset{\sim}{v})) = M_*(1,1,1,p,1\emptyset(\underset{\sim}{v})) = M^*(1,1,1,p,2\emptyset(\underset{\sim}{v})) = M_*(1,1,1,p,2\emptyset(\underset{\sim}{v})).$$

In the ensuing computations we make use of the following easily established

Lemma: If $P_1 = P_2$ with probability one and they have common marginal c.d.f. F_0, then

$$U_F(s,t) = V_F(s,t) = \begin{cases} 0 & \text{if } s \geq t \\ \int_s^t w dF_0(w) & \text{if } s < t. \end{cases}$$

Back to our example, we use this lemma and the equations of sections 4 and 7 to obtain

(9.2) $\mu^*(1,1,0,\emptyset(\underset{\sim}{v})) = 1,$

(9.3) $\mu^*(1,0,1,\emptyset(\underset{\sim}{v})) = u^*(1,1) = u^*(0,1) + u^*(1,0)T_{F_1}\left(\frac{u^*(0,1)}{u^*(1,0)}\right) =$

$$\tfrac{1}{2} + \int_{\frac{1}{2}}^{1}(x-\tfrac{1}{2})dx = \frac{5}{8},$$

(9.4) $\mu^*(0,1,1,\emptyset(\underset{\sim}{v})) = v^*(1,1) = v^*(0,1)[1-F_2(\frac{v_*(0,1)}{v_*(1,0)})] +$

$$v^*(1,0)U_F(0,\frac{v_*(0,1)}{v_*(1,0)}) = \tfrac{1}{2}[1-F_2(\tfrac{1}{2})] + U_F(0,\tfrac{1}{2}) = \frac{1}{4} + \int_0^{\frac{1}{2}} w dw = \frac{3}{8},$$

and hence from (4.2) we obtain

(9.5) $M^*(1,1,1,p,1\emptyset(\underset{\sim}{v})) = \begin{cases} 3/8 & \text{if } p < 3/8 \\ p & \text{if } 3/8 < p < 5/8 \\ 5/8 & \text{if } p > 5/8. \end{cases}$

The equalities (9.1) can be verified by computations similar to the above for the other M^* and M_* functions. It is easily found from either (9.5) or (7.4) that

(9.6) $\mu^*(1,1,1,1\emptyset(\underset{\sim}{v})) = \tfrac{1}{2}[= \mu_*(1,1,1,1\emptyset(\underset{\sim}{v})) = \mu^*(1,1,1,2\emptyset(\underset{\sim}{v})) =$

$$\mu_*(1,1,1,2\emptyset(\underset{\sim}{v}))].$$

Hence both sides start with expectation $\frac{1}{2}$. The optimal strategy is as follows: The first candidate will be accepted if his p-value is between $\frac{3}{8}$ and $\frac{5}{8}$; otherwise he will be challenged by the appropriate party. If either side challenges the first juror, his opponent will challenge the second juror if and only if he finds $\frac{1}{2}$ preferable to this second p-value. If the defense uses the first challenge, the expectation for both sides becomes $\frac{5}{8}$; if the prosecution uses the first challenge, this common expectation is then $\frac{3}{8}$. Of course, the mutual expectation returns to $\frac{1}{2}$ if both sides use their challenges.

Example 2: P_1 and P_2 are independent. By Class 1 of section 8, there is no reversibility (so that we must compute four different values of M^* or M_* and four different values of μ^* or μ_*). It can be shown that (for $0<x<1$, $0<y<1$)

$$(9.7) \qquad U_F(x,y) = \tfrac{1}{2}\, y(1-x^2) \quad \text{and} \quad V_F(x,y) = \tfrac{1}{2}\, y^2(1-x).$$

Using (9.7) and the equations of sections 4 and 7, we obtain

$$(9.8)\ \mu^*(0,1,1,\emptyset(\underset{\sim}{y})) = v^*(1,1) = v^*(0,1)\left[1-F_2\left(\frac{v_*(0,1)}{v_*(1,0)}\right)\right] + v^*(1,0)U_F\left(0,\frac{v_*(0,1)}{v_*(1,0)}\right) = \tfrac{1}{2}\left[1-F_2(\tfrac{1}{2})\right] + U_F(0,\tfrac{1}{2}) = \tfrac{1}{2},$$

$$(9.9)\ \mu^*(1,0,1,\emptyset(\underset{\sim}{y})) = u^*(1,1) = u^*(0,1) + u^*(1,0)T_{F_1}\left(\frac{u^*(0,1)}{u^*(1,0)}\right) = \tfrac{1}{2} + T_{F_1}(\tfrac{1}{2}) = \frac{5}{8},$$

$$(9.10)\ \mu^*(1,1,0,\emptyset(\underset{\sim}{y})) = \mu_*(1,1,0,\emptyset(\underset{\sim}{y})) = 1,$$

(9.11) $u_*(0,1,1,\emptyset(\underset{\sim}{v})) = v_*(1,1) = \frac{1}{2}v_*(1,0) - v_*(1,0)T_{F_2}\left(\frac{v_*(0,1)}{v_*(1,0)}\right) =$

$$\tfrac{1}{2} - T_{F_2}(\tfrac{1}{2}) = \frac{3}{8}, \text{ and}$$

(9.12) $u_*(1,0,1,\emptyset(\underset{\sim}{v})) = u_*(1,1) = u_*(0,1)F_1\left(\frac{u^*(0,1)}{u^*(1,0)}\right) +$

$$u_*(1,0)V_F\left(\frac{u^*(0,1)}{u^*(1,0)},1\right) = \tfrac{1}{2}F_1(\tfrac{1}{2}) + V_F(\tfrac{1}{2},1) = \tfrac{1}{2}.$$

These equations allow us to use section 4 to obtain the following table for the optimal strategy (and the M^* and M_* values) regarding the first prospective juror:

(9.13)

$M^*(1,1,1,p_1,p_2,1\emptyset(\underset{\sim}{v}))$

	$p_1 < \frac{1}{2}$	$p_1 > \frac{1}{2}$
$p_2 < \frac{1}{2}$	$\frac{1}{2}$ ***	p_1 *
$p_2 > \frac{1}{2}$	$\frac{5}{8}$ **	$\frac{5}{8}$ **

$M_*(1,1,1,p_1,p_2,1\emptyset(\underset{\sim}{v}))$

	$p_1 < \frac{1}{2}$	$p_1 > \frac{1}{2}$
$p_2 < \frac{1}{2}$	$\frac{3}{8}$ ***	p_2 *
$p_2 > \frac{1}{2}$	$\frac{1}{2}$ **	$\frac{1}{2}$ **

$M^*(1,1,1,p_1,p_2,2\emptyset(\underset{\sim}{v}))$

	$p_1 < \frac{1}{2}$	$p_1 > \frac{1}{2}$
$p_2 < \frac{1}{2}$	$\frac{1}{2}$ ***	p_1 *
$p_2 > \frac{1}{2}$	$\frac{1}{2}$ ***	$\frac{5}{8}$ **

$M_*(1,1,1,p_1,p_2,2\emptyset(\underset{\sim}{v}))$

	$p_1 < \frac{1}{2}$	$p_1 > \frac{1}{2}$
$p_2 < \frac{1}{2}$	$\frac{3}{8}$ ***	p_2 *
$p_2 > \frac{1}{2}$	$\frac{3}{8}$ ***	$\frac{1}{2}$ **

* = both accept, ** = defense challenges, *** = prosecution challenges

Note that each person can expect $\frac{1}{2}$ if he uses his challenge since his opponent's strategy is independent of his own perception of the p-values. The optimal strategy beyond the first juror is the same as in Example 1 since each side's strategy when his opponent is out of challenges depends only on the appropriate marginal distribution and not on the joint distribution of P_1 and P_2.

To see how much is gained by going first on the first juror, we use (7.4) through (7.7) to compute

$$(9.14)\quad \mu^*(1,1,1,1\emptyset(\underset{\sim}{v})) = \tfrac{1}{2}F(\tfrac{1}{2},\tfrac{1}{2}) + \frac{5}{8}[1-F_2(\tfrac{1}{2})] + U_F(\tfrac{1}{2},\tfrac{1}{2}) = \frac{5}{8},$$

$$(9.15)\quad \mu^*(1,1,1,2\emptyset(\underset{\sim}{v})) = \tfrac{1}{2}F_1(\tfrac{1}{2}) + \frac{5}{8}\,\overline{F}(\tfrac{1}{2},\tfrac{1}{2}) + U_F(\tfrac{1}{2},\tfrac{1}{2}) = \frac{19}{32} = \frac{5}{8} - \frac{1}{32},$$

$$(9.16)\quad \mu_*(1,1,1,1\emptyset(\underset{\sim}{v})) = \frac{3}{8}\,F(\tfrac{1}{2},\tfrac{1}{2}) + \tfrac{1}{2}[1-F_2(\tfrac{1}{2})] + V_F(\tfrac{1}{2},\tfrac{1}{2}) = \frac{13}{32}, \text{ and}$$

$$(9.17)\quad \mu_*(1,1,1,2\emptyset(\underset{\sim}{v})) = \frac{3}{8}\,F_1(\tfrac{1}{2}) + \tfrac{1}{2}\,\overline{F}\,(\tfrac{1}{2},\tfrac{1}{2}) + V_F(\tfrac{1}{2},\tfrac{1}{2}) = \frac{3}{8} = \frac{13}{32} - \frac{1}{32};$$

Thus each side can expect to do $\frac{1}{32}$ better by going first than by going second.

References

1. Arnold, Martin, "How Mitchell-Stans Jury Reached Acquittal Verdict," NYT, May 5, 1974, p. 1ff.

2. Cyert, R. M. and M. H. DeGroot (1970), "Multiperiod Decision Models with Alternating Choice as a Solution to the Duopoly Problem," Quart. J. Econ., August, 1970, 84, pp. 410-429.

3. Cyert, R. M. and M. H. DeGroot (1973), "An Analysis of Cooperation and Learning in a Duopoly Context," Amer. Econ. Review, March, 1973, LXIII, pp. 24-37.

4. DeGroot, M. (1970), Optimal Statistical Decisions, McGraw-Hill, New York.

5. Etzioni, Amitai, "Science: Threatening the Jury Trial," Washington Post, May 26, 1974, Sec. C, p. 3, col. 1.

6. Ginger, A. F. (1975), Jury Selection in Criminal Trials, Lawpress Corp.: Tiburon, Calif.

7. Kairys, D. (1975), The Jury System: New Methods for Reducing Prejudice, Philadelphia Resistance Print Shop: Philadelphia, Pa.

8. Schulman, J. et al., "Recipe for a Jury," Psychology Today, May, 1973, 37ff.

9. Shapley, D., "Jury Selection: Social Scientists Gamble in an Already Loaded Game," Science 185: 1033-4, 1071 S 20, 74.

10. Zeisel, H. (1974), "Mitchell-Stans Judged," NYT, May 26, p. E15.

11. Zeisel, H. and S. S. Diamond (1976), "Jury Selection in the Mitchell-Stans Conspiracy Trial," American Bar Foundation Research Journal 1, pp. 151-174.

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Prosecution and defense lawyers are about to select a jury of J people. Each prospective juror is (sequentially) interviewed, and each lawyer must then decide whether to accept or challenge (i.e., reject) the present candidate before interviewing anyone else, and this decision cannot later be changed. (over)

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The prosecution is allowed at most A challenges while the defense has at most B of them. After questioning each juror, the two sides have (possibly different) opinions about the probability that this person will vote to convict the defendant, giving rise to a vector (p_{1i}, p_{2i}) of the opinions of the prosecution and the defense, respectively. The joint c.d.f. $F(p_1, p_2)$ of the bivariate random variable (P_1, P_2) throughout the population is assumed known, so that the observed values (p_{1i}, p_{2i}) represent a sequential random sample from F. It is also assumed that after a juror is questioned, each side knows both its own and its opponent's opinion, i.e., the questioning process gives both sides simultaneously a complete (bivariate) observation from F. Furthermore, the rule determining at each stage which side must specify first whether it wishes to use a challenge is assumed fixed at the outset and does not depend on the previous decisions of the participants. We also assume that using its peremptive challenges the prosecution wishes to maximize $\prod_{i=1}^{J} p_{1i}$, while the defense using its peremptive challenges wishes to minimize $\prod_{i=1}^{J} p_{2i}$, where the products are taken over the members of the jury actually chosen. Properties of the optimal strategies are studied, and particular attention is paid to the case of reversibility in which neither side cares who challenges first. While phrased in terms of juror selection, the same principles apply to any bilateral sequential decision process.

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